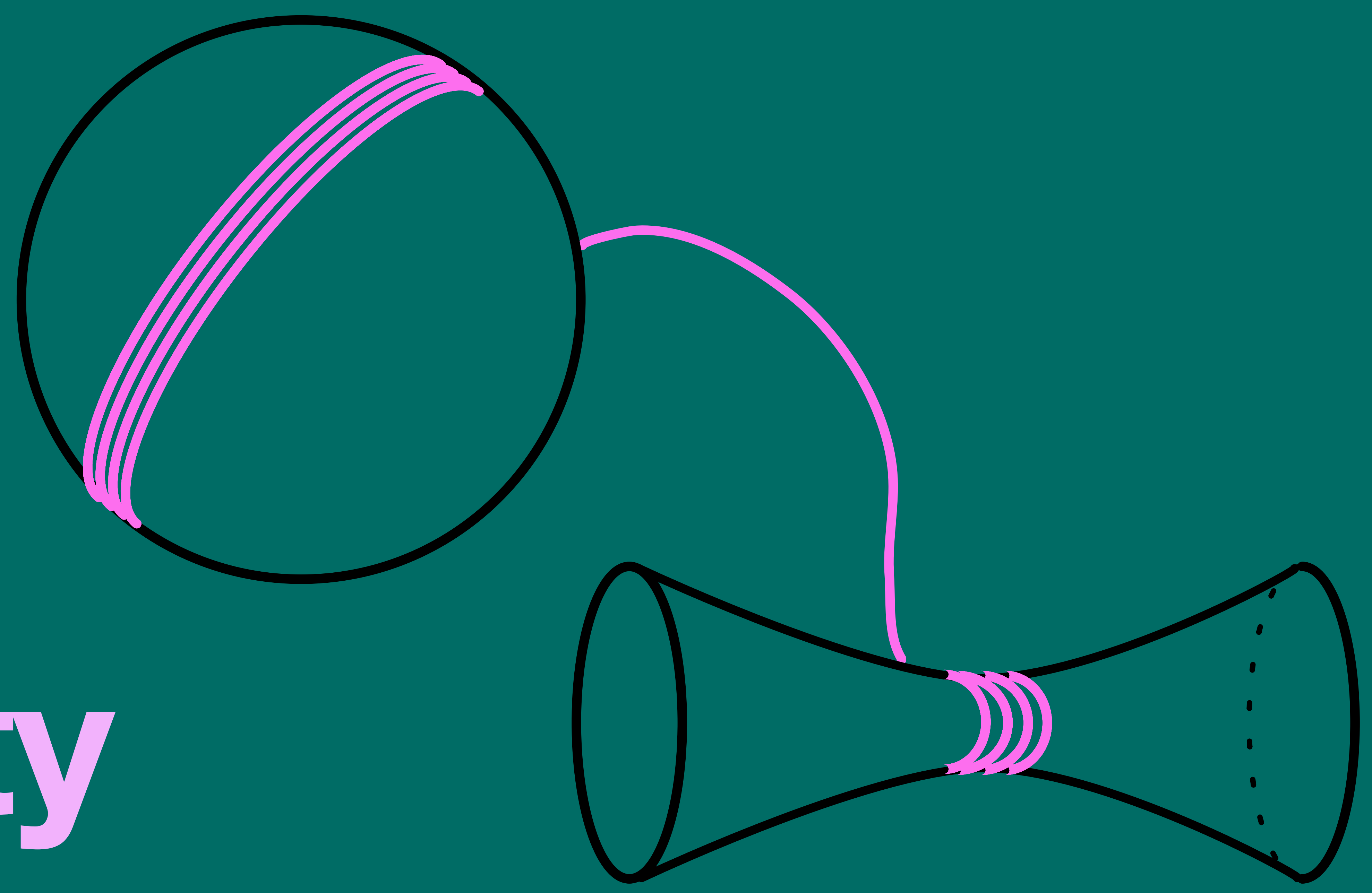
Jackson R. Fliss

Keeping matter in the loop in 3d quantum gravity



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w/ A. Castro, I. Coman, C. Zukowski

@ Yukawa Institute of Theoretical Physics
ExU-YITP QIMG 2023

2 October 2023

UNIVERSITY OF CAMBRIDGE

New perspectives on gravity

- ***Gravity from …***
  - quantum entanglement
  - quantum complexity
  - chaotic dynamics
  - …
  - … as a “mongrel” classical / quantum hybrid?…

Gravity and order

- I want to make some connection to some themes from the previous week

- I want to focus on a different aspect of gravity: ***long-range*** ***order***
  - Non-local (dressed) observables
  - Constraints $\longrightarrow$ Drastic reduction in degrees of freedom
  - On compact spatial slices, constraints can be very strong

[Chakraborty, Chakravarty, Godet, Paul, Raju] [Shaghoulian]

Indications of "finite features" in de Sitter spacetime?

[Anninos, Galante, Mülmann]

3d gravity and Chern-Simons theory

- This analogy is made manifest in ***three space-time dimensions***
  - 3d gravity has no propagating d.o.f: ***topological field theory***

$$\boxed{\mathcal{Z}_{\mathrm{grav}} \simeq Z_{\mathrm{CS}}\, Z_{\mathrm{CS}}}$$

[Achucarro, Townsend; ’86]
[Witten, ’88]

- We are very comfortable with the story in ***Anti-de Sitter spacetime*:**
  - Pure gravity: $SL(2,\mathbb{R})_L \times SL(2,\mathbb{R})_R$
  - Successful framework for discussing BHs and higher-spin extensions.
  - **holographic dictionary ~ bulk / edge correspondence** (with app. BCs)

- However also a particularly useful approach for ***de Sitter*** space
  - Euclidean dS_3 is compact
  - Pure gravity: $SU(2) \times SU(2)$
  - Many (exact) techniques for these types of Chern-Simons theories
  - **Saddle-by-saddle**: match graviton determinants at 1-loop
  - *(sum over saddles: your mileage may vary….)*

[Carlip,92]
[Govindarajan, Kaul, Sunneta; '02]
[Castro, Lashkar, Maloney, '11]
[Anninos, Denef, Law, Sun, '22]
[Hikida, Nishioka, Takayanagi, Taki; '22]
…

[Maloney, Witten; '07]
[Castro, Lashkari, Maloney; '11]

In this talk: $$Z_{\text{grav}}[M] = Z_{\text{CS}}[M] Z_{\text{CS}}[M]$$

fixed saddle *fixed topology*

How do we incorporate matter while utilizing LRO?

- We would like QG to compute more than one number, $\mathcal{Z}_{\rm grav}$
- *… or … How does quantum gravity alter the physics of quantum matter?*
- ***Metric language:***
    - manifest locality, spacetime geometry explicit
    - minimally couple with metric compatible ∇_μ

$$Z_{\rm scalar}[g_{\mu\nu}] = \int [D\phi] e^{-S_{\rm scalar}[\phi, g_{\mu\nu}]}$$

- ***CS language:***
    - manifest diffeo invariance; all-loop exactness
    - *How to couple matter while maintaining these features?*

- ***Euclidean gravity as a low-energy effective theory***
  - CS ~ EFT from integrating out matter above some mass gap

Worldline of charged particle ~ Wilson Line

  - *The goal of this talk is to make this intuition exact.*

- **Our proposal:**

$$\boxed{\log Z_{\text{scalar}}[g_{\mu\nu}] = \frac{1}{4}\mathbb{W}_j[A_L, A_R]}$$

- $\mathbb{W}_j$ = ***"Wilson spool"***

$$\boxed{\mathbb{W}_j = i\int_{\mathcal{C}} \frac{d\alpha}{\alpha}\frac{\cos\alpha/2}{\sin\alpha/2}\mathrm{Tr}_{R_j}\left(Pe^{\frac{\alpha}{2\pi}\oint A_L}\right)\mathrm{Tr}_{R_j}\left(Pe^{-\frac{\alpha}{2\pi}\oint A_R}\right)}$$

Orienting ourselves with main result

$$\boxed{\mathbb{W}_j = i\int_{\mathcal{C}} \frac{d\alpha}{\alpha}\frac{\cos\alpha/2}{\sin\alpha/2}\mathrm{Tr}_{R_j}\left(Pe^{\frac{\alpha}{2\pi}\oint A_L}\right)\mathrm{Tr}_{R_j}\left(Pe^{-\frac{\alpha}{2\pi}\oint A_R}\right)}$$

Connections $\rightarrow$ background geometry, $g_{\mu\nu}$

Representations $\rightarrow$ mass / spin of field

Contour forces winding:

$$\text{“}= \sum_{n=1} \frac{1}{n}\mathrm{Tr}_{R_j}\left(Pe^{n\oint A_L}\right)\mathrm{Tr}_{R_j}\left(Pe^{-n\oint A_R}\right)\text{”}$$

[Ooguri, Vafa; 99]

***NB:** This is literally true for **BTZ** geometry. In **dS** the pole structure is more interesting.*

- ***Tree-level checks***

AdS

$$\boxed{\lim_{G_N \to 0} \left\langle \mathbb{W}_j \right\rangle = \log Z_{\text{scalar}}[\text{BTZ}]}$$

dS

$$\boxed{\lim_{G_N \to 0} \left\langle \mathbb{W}_j \right\rangle = \log Z_{\text{scalar}}[S^3]}$$

  - *NB: exact logZ, i.e. no large mass / geodesic approx.*

- These are non-trivial tests, however I should emphasize that the expression for the Wilson spool *can be derived:*
  - log Z as product over quasi-normal mode spectra *[Denef, Hartnoll, Sachdev; '10]*
  - organize spectra in $\mathfrak{sl}(2)$ / $\mathfrak{su}(2)$ representation theory

For the sake of concreteness / time, this talk will focus on construction in **de Sitter**

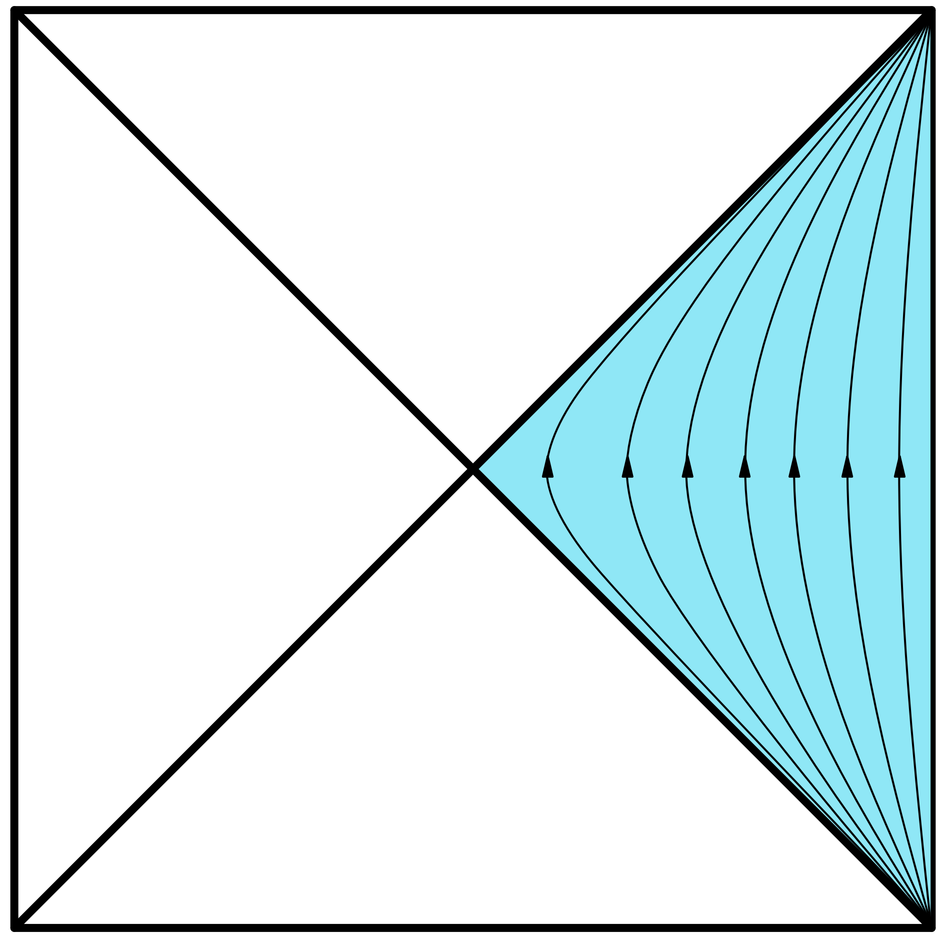
(however framework applies just as well to AdS)

- In **de Sitter**: ***calculational control beyond tree-level***
  - "exact methods" (e.g Abelianisation) reduce $\left\langle \mathbb{W}_R \right\rangle$ to an ordinary integral.
  - Can be performed in G_N perturbation theory and is *finite* at each order.
  - Concrete predictions for renormalizing matter coupled to QG.

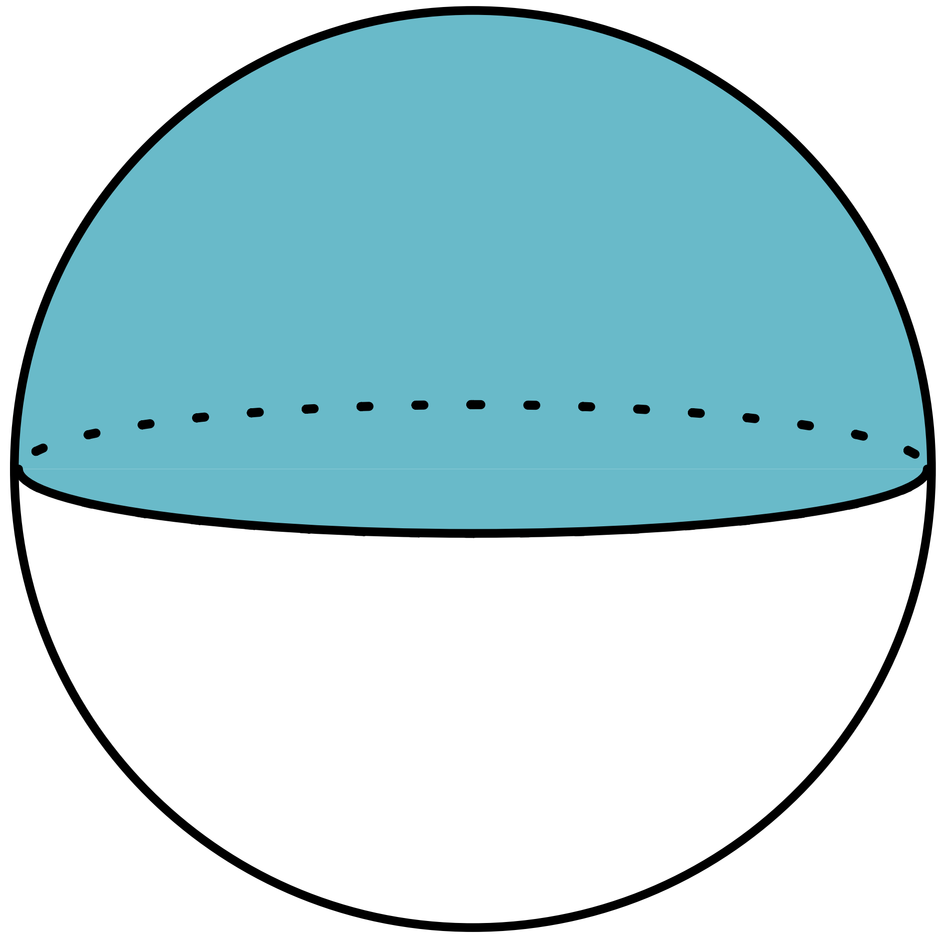
dS_3 Chern-Simons gravity

dS_3

Expanding spatial slices → observers inside causal horizons



$$\to \quad t = -i\tau \quad \to$$



$\simeq S^3$

- dS Wick rotates to a sphere.
- Isometry group: $SO(4) \simeq SU(2) \times SU(2)/\mathbb{Z}_2$
- Smoothness at horizon, $\tau \sim \tau + 2\pi$

Chern-Simons gravity: the dictionary

$$SU(2)_L \quad \times \quad SU(2)_R$$

$$S = \frac{k_L}{4\pi}\mathrm{Tr}\int \left(A_L \wedge dA_L + \frac{2}{3}A_L^3\right) + \frac{k_R}{4\pi}\mathrm{Tr}\int \left(A_R \wedge dA_R + \frac{2}{3}A_R^3\right)$$

$k_L = \delta + is \qquad k_R = \delta - is \qquad \delta \in \mathbb{Z} \quad s \in \mathbb{R}$ *[Witten,88] [Witten,91]*

$$\boxed{iS = -\frac{1}{16\pi G_N}\int (R - 2\Lambda) + i\frac{\delta}{2\pi}I_{\mathrm{GCS}}} \qquad \boxed{s = \frac{\ell}{4G_N}}$$

NB: semi-classical limit,
$G_N \to 0$ = $s \to \infty$

See [Hikida, Nishioka, Takayanagi, Taki; '22] for alternative limit

$$A_L = i\,(\omega^a + e^a/\ell)\,L_a \qquad\qquad A_R = i\,(\omega^a - e^a/\ell)\,\bar{L}_a$$

spin-connection *frame*

- Finding an appropriate flat background $a_{L/R}$:

$$g^{S^3}_{\mu\nu} = \delta_{ab} e^a_\mu e^b_\nu \quad \to \quad e^a \quad \to \quad \omega^a \quad \to \quad a_{L/R}$$

- e (and *g*) are well defined, but $a_{L/R}$ may possess singularities

$$\boxed{ik_L S_{\mathrm{CS}}[a_L] + ik_R S_{\mathrm{CS}}[a_R] = 2\pi s = \frac{\pi\ell}{2G_N} = S_{\mathrm{dS}}}$$

- Important: they also have holonomy around these points

$$\mathcal{P}\exp\left(\oint_\gamma a_{L/R}\right) \sim e^{i2\pi \mathsf{h}_{L/R} L_3}$$

the gravitational path-integral

- Modify "exact methods" for when $a_{L/R} \neq 0$ and $k_{L/R} \in \mathbb{C}$
  - *Abelianisation, $\mathcal{N} = 2$ SUSY localization*

- Expression of $\mathcal{Z}_{\text{grav}}$ as an ordinary integral (which is easily performed)

$$\boxed{\mathcal{Z}_{\text{grav}} = i e^{2\pi s} \frac{2}{\sqrt{r_L r_R}} \sin\left(\frac{\pi}{r_L}\right) \sin\left(\frac{\pi}{r_R}\right) = i e^{S_{\text{dS}}} \left(\frac{8G_N}{\ell}\right) \sinh^2\left(\frac{4\pi G_N}{\ell}\right)}$$

[Carlip,92] [Govindarajan, Kaul, Sunneta; '02]
[Castro, Lashkari, Maloney; '11]
[Anninos, Denef, Law, Sun; '22]
[Hikida, Nishioka, Takayanagi, Taki; '22]…

NB: finite renormalization of the level:

$$k_{L/R} \to r_{L/R} = k_{L/R} + 2$$

Looping matter in

minimally coupled scalar fields

- Laplacian is quadratic Casimir of isometry group:

$$\frac{1}{4}\nabla^2 = c_2 = -\frac{m^2\ell_{\mathrm{dS}}^2}{4}$$

- So we need highest-weights, j , that are continuous or even complex!

$$\boxed{j = -\frac{1}{2}\left(1+\sqrt{1-m^2\ell^2}\right)}$$

$\xrightarrow{m^2\ell^2<1}$ $\boxed{j = -\frac{1}{2}(1+\nu) \qquad \nu \in (-1,1)}$ *"complimentary -type"*

$\xrightarrow{m^2\ell^2>1}$ $\boxed{j = -\frac{1}{2}(1+i\mu) \qquad \mu \in \mathbb{R}}$ *"principal-type"*

- ∞ -dim'l highest weight representations of $\mathfrak{su}(2)$ $\longrightarrow$ QNM spectra of dS_3

how do we include this matter?

- Recall EFT intuition: **Wilson loops ~ worldlines of massive particles**
- **Worldline QM**: sum over worldlines. Include wrapping on compact directions.
- Expect collection of wrapped Wilson loops ~ $\log Z_{\mathrm{scalar}}$.

[Ooguri, Vafa; '99]

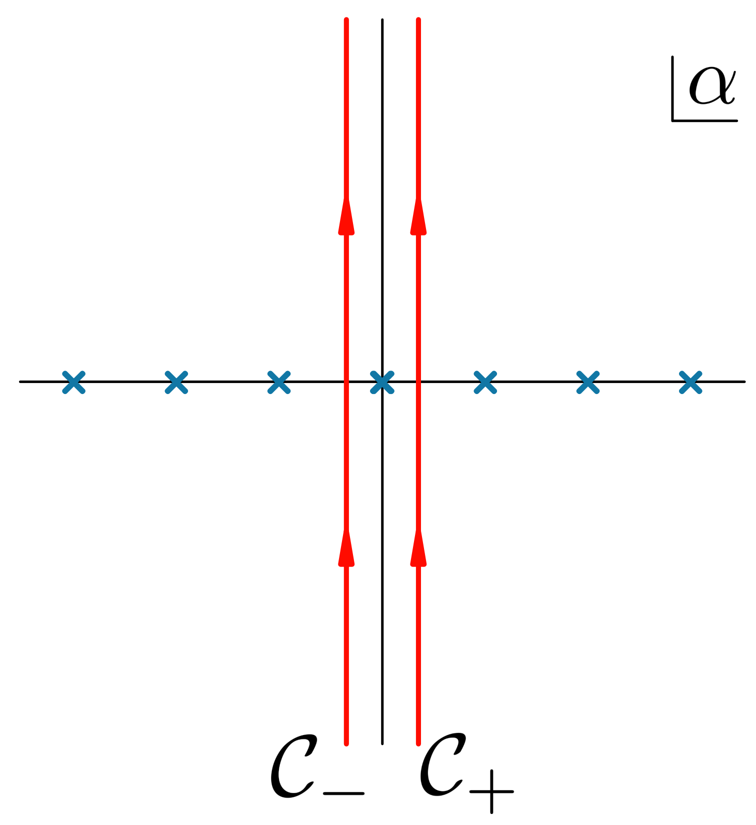
- However we need a more **systematic construction** to correctly apply to QG

Apply QNM physics in su(2) natural language

[Denef, Hartnoll, Sachdev; '10]

- Let's start with S^3 and generalize afterward
- Z_{scalar} ~ a meromorphic function of mass: match to function w/ same zeros/poles
- $Z_{\text{scalar}} = \det(-\nabla^2 + m^2\ell^2)^{-1/2}$ has poles for states w/ $c_2 = -\frac{m^2\ell^2}{4}$
    - ***"Non-standard" reps we constructed earlier!***
- ***Smoothness at horizon:*** A weight $(\lambda_L, \lambda_R) \in R_j \times R_j$ can contribute a pole when parallel transport around the horizon is periodic.
- **DHS logic**: Z_{scalar} = product over weights of $R_j \times R_j$ subject to quantization condition on **weight x holonomy**
- Taking a log organizes this product into sum $\longrightarrow$ representation trace.

thus appears the Wilson spool



$$\boxed{\log Z_{\text{scalar}}[S^3] = \frac{1}{4}\mathbb{W}_j[a_L, a_R]}$$

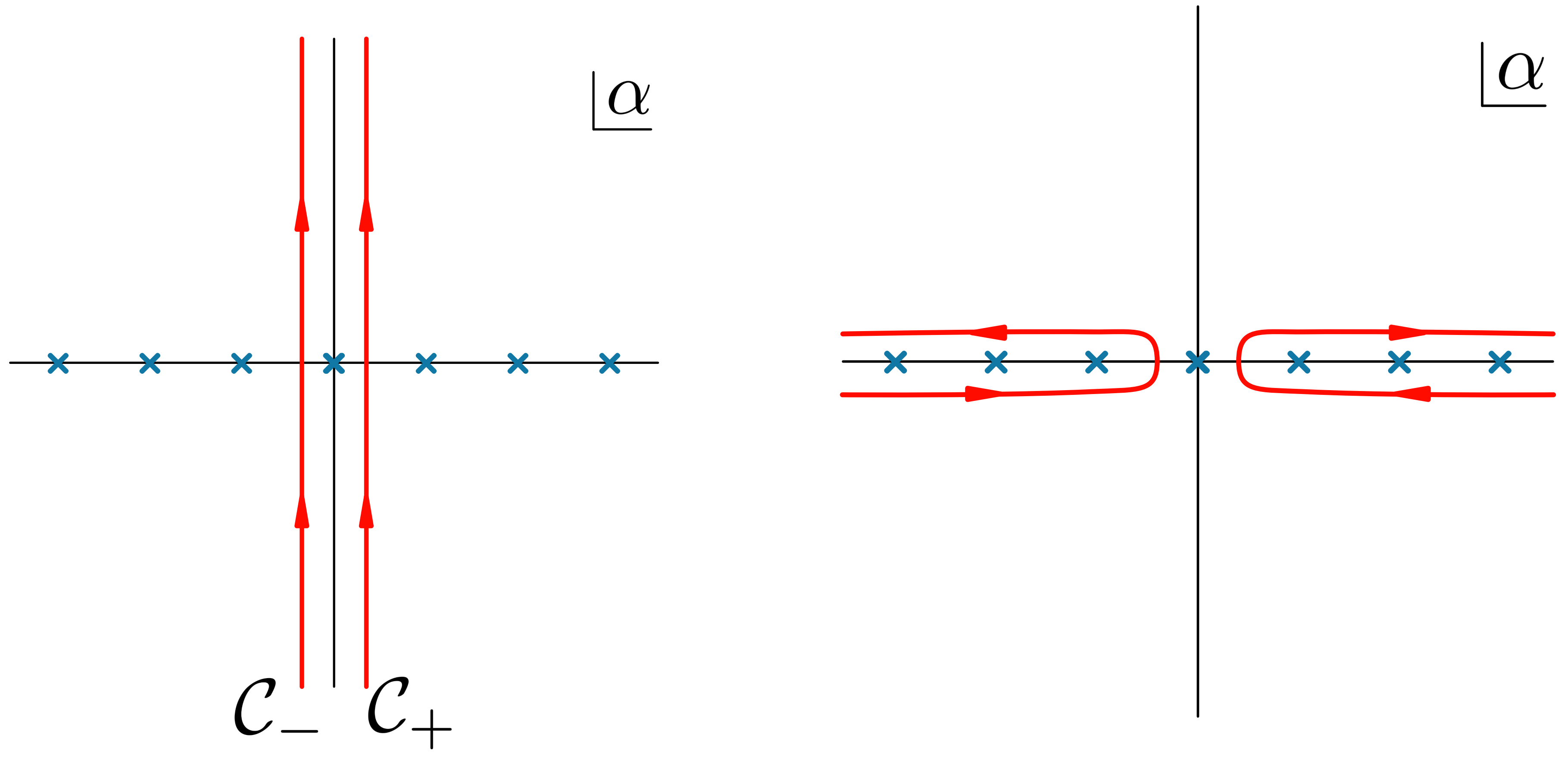
$$\boxed{\mathbb{W}_j = i\int_{\mathcal{C}} \frac{d\alpha}{\alpha}\frac{\cos\alpha/2}{\sin\alpha/2}\mathrm{Tr}_j\left(\mathcal{P}\exp\left(\frac{\alpha}{2\pi}\oint_\gamma a_L\right)\right)\mathrm{Tr}_j\left(\mathcal{P}\exp\left(-\frac{\alpha}{2\pi}\oint_\gamma a_R\right)\right)}$$

- Expresses log*Z* as integral over gauge invariant WL operators
- One can show that it generalizes: when $A_{L/R}$ define a metric geometry, M_3

$$\boxed{\log Z_{\text{scalar}}[M_3] = \frac{1}{4}\mathbb{W}_j[A_L, A_R]}$$

Test 1: Classical dS (*G*$_N$ ⟶ *0*)

- Wrap α integral around poles at $\pm 2\pi n$ and sum residues



$$\frac{1}{4}\mathbb{W}_j[a_L, a_R]$$

$$= \frac{\pi\mu^3}{6} - \frac{1}{4\pi^2}\sum_{q=1}^{3} \mathrm{Li}_q\left(e^{-2\pi\mu}\right)\frac{(2\pi\mu)^{q-3}}{(q-3)!}$$

$$\mu = \sqrt{m^2\ell^2 - 1}$$

$$= \log Z_{\text{scalar}}[S^3]$$ ✔

- Appearance of polylogarithms = sums over winding world-lines
  - *(tracks similar calculations of heat kernel as world-line QM)*
- *This computation is **completely finite***: no need to minimally subtract divergences from log Z_{scalar}

quantum gravity and quantum matter

- Let us insert in gravitational path-integral

$$\Big\langle \log Z_{\text{scalar}} \Big\rangle_{\text{grav}} = \int \mathcal{D}g_{\mu\nu} \log Z_{\text{scalar}}[g] e^{-I_{\text{EH}}[g]} = \frac{1}{4} \int \mathcal{D}A_L \mathcal{D}A_R \, \mathbb{W}_j[A_L, A_R] e^{iS_{\text{CS}}}$$

- **Abelianisation:** this expectation value reduces to ordinary integrals over representation characters
  - integral can be computed in G_N perturbation theory: *Finite at each order*
  - e.g. we compute corrections to $\log Z_{\text{scalar}}$ to order $\mathcal{O}(G_N^2)$

Predictive statement about how matter is renormalized in QG

$$\mu_{\text{bare}} = \mu_{\text{renorm}} + \frac{48}{5} \mu_{\text{renorm}}^3 \, e^{-2\pi \mu_{\text{renorm}}} \left(\frac{G_N^2}{\ell^2} \right) + \dots$$

Recap and outlook

- ***3d QG has features of topological order*** $\longrightarrow$ *exact evaluation of PI*
- ***How to incorporate matter while respecting this order?***
- ***Result: Wilson spool*** captures the effective physics of log Z_{scalar}
    - DHS expression of Z_{scalar} using modified su(2) reps of QNM spectra
    - Reproduces 1-loop det as G_N vanishes (***three-sphere*** ✓)
    - Abelianisation $\longrightarrow$ finite computation of higher G_N corrections
        - $\longrightarrow$ ***predictions***

future applications?

- **Including matter into the "Farey tail" sum over topologies** *[Maloney, Witten], [Castro, Lashkari, Maloney]*
    - ***does the non-perturbative divergence persist?***
- **Quantum corrected dS_3 black holes** *[Emparan, Pedraza, Svesko, Tomasevic, Visser]*
    - ***CS formalism $\longrightarrow$ higher-spin extensions?***
- **Interpretations in terms of edge modes / dS holography?** *[Hikida, Nishioka, Takayanagi, Taki; '22]*
    - ***Topological bulk / edge correspondence***
    - ***Where do our representations fit? What do they say about the unitarity of the dual theory?***

Thank you for your attention.

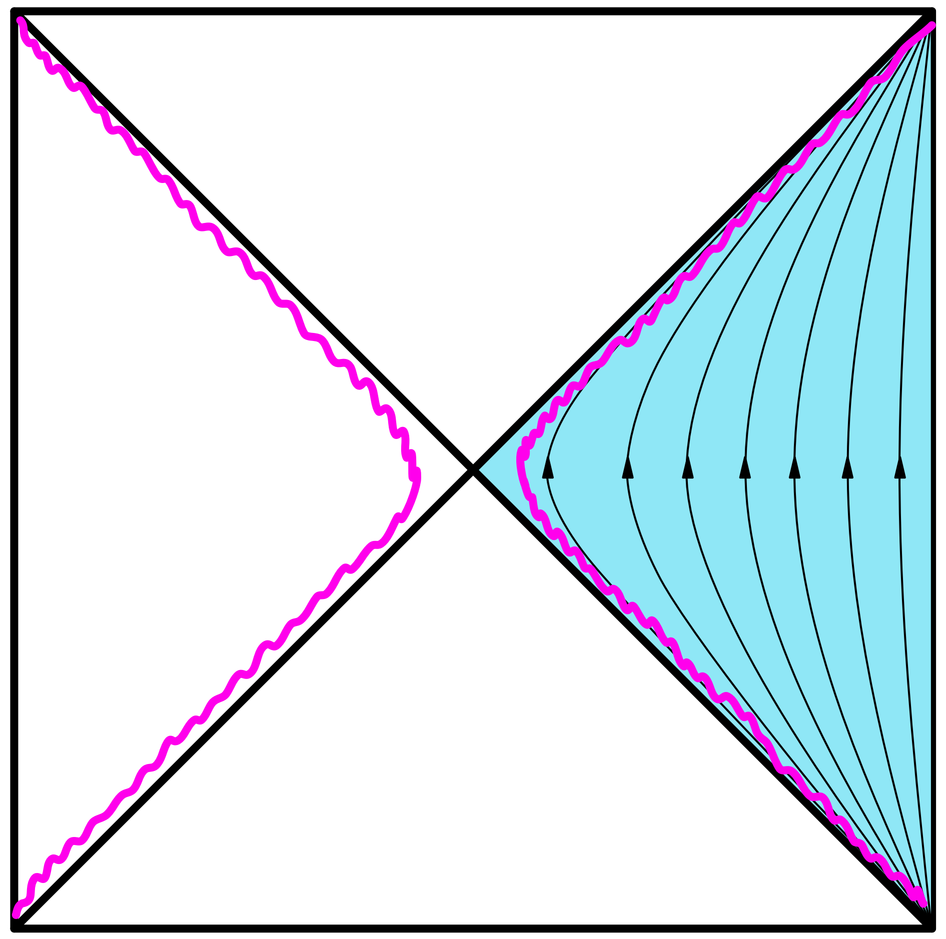
$\Big\langle \log Z \Big\rangle_{\text{grav}}$ vs. $\log \Big\langle Z \Big\rangle_{\text{grav}}$

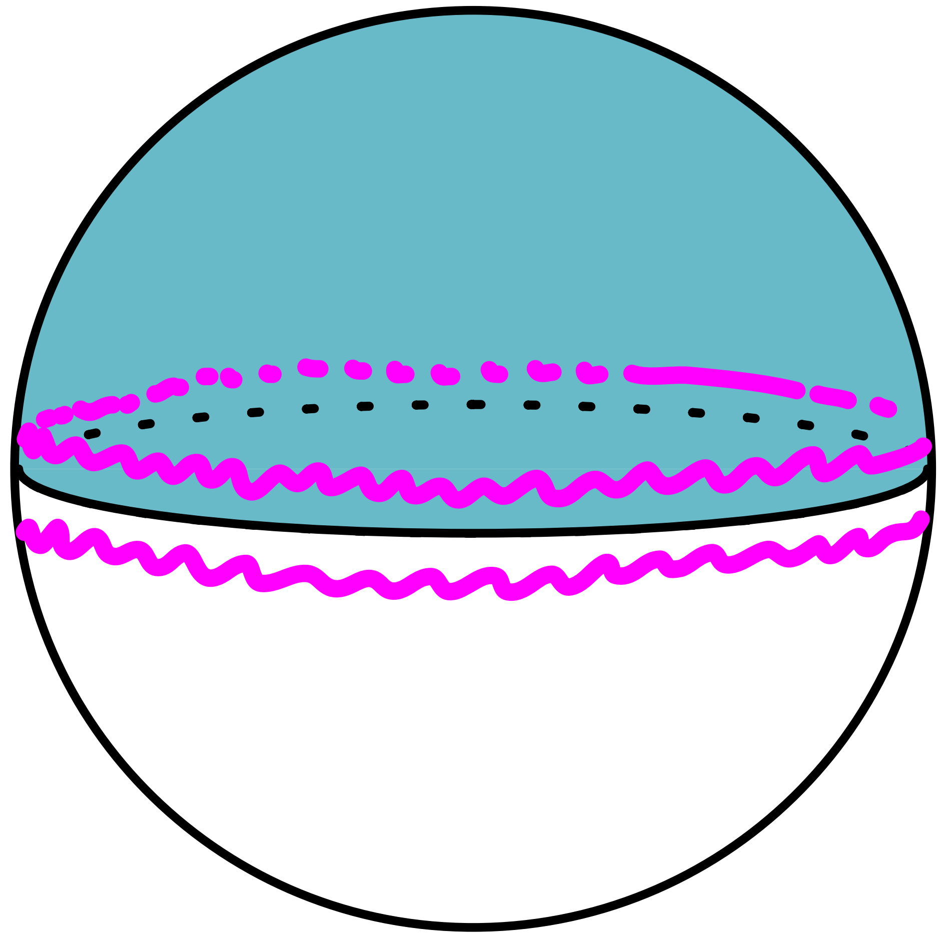
- Normalizing EVs by $\mathcal{Z}_{\text{grav}}^{-1}$, these two agree at first non-trivial order, $G_N^2 \ell^{-2}$
- …but generically different (think *quenched vs. annealed disorder*)
- However former is straightforward but latter is likely ***hard*** to compute.
    - $Z_{\text{scalar}} = e^{\frac{1}{4}\mathbb{W}_j}$ ~ arbitrary products of Wilson loop ops
    - These should be allowed to link: still need systematic, efficient method for evaluating EV.

$$\sum_n \frac{1}{n!} \sum_{n \text{ links}} \Big\langle \mathbb{W}_j[\gamma_1]\mathbb{W}_j[\gamma_2]\ldots\mathbb{W}_j[\gamma_n] \Big\rangle \stackrel{?}{=} e^{-F_{\text{grav+matter}}}$$

Edge modes & dS/CFT

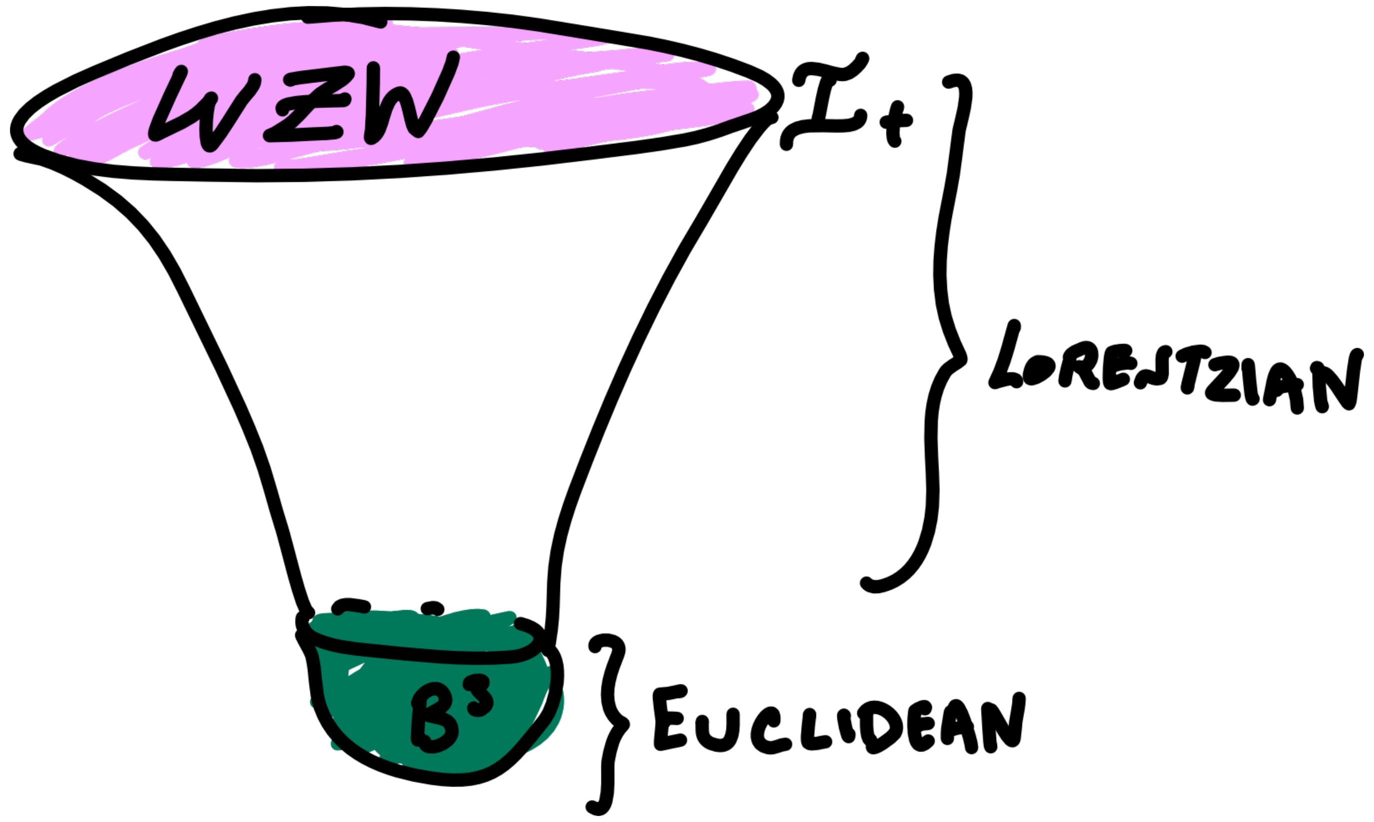
- **LRO :** Bulk / Edge correspondence of topological phases.





Exploring dS/CFT dictionary?

- Unitarity properties of dual?
- dS entropy as entanglement?
- Other explorations:
  - complexity,
  - local probes = Wilson lines anchored to stretched horizon
  - …



[Hikida, Nishioka, Takayanagi, Taki; ‘21+22]

More ramblings on 3d dS / CFT

- Spectrum of Wess-Zumino-Witten model generated by $\hat{\mathfrak{su}}(2)_k$ current algebra

$$[J^a_m, J^b_n] = i\varepsilon^{abc} J^c_{m+n} + k\, m \delta_{m+n} \delta^{ab}$$

- Standard rep, $\hat{R}$, of affine algebra built from rep, *R*, of finite algebra
  - Unitarity of *R* is *not a guarantee of unitarity of* $\hat{R}$ *!*
    - *see e.g.* $\hat{\mathfrak{sl}}(2,\mathbb{R})_k$ *[Dixon, Peskin, Lykken] [Maldacena, Ooguri]*
  - In this case $k = \delta \pm is \in \mathbb{C}$ gives us problems even for $\hat{\mathfrak{su}}(2)_k$
    - Easy to find states $\Big| |\psi\rangle \Big|^2 \propto k$

Fundamental non-unitarity? Perhaps need to alter inner product?